

INTERNATIONAL
STANDARD

ISO/CIE
11664-4

First edition
2019-06

Colorimetry —
Part 4:
CIE 1976 $L^*a^*b^*$ colour space

Colorimétrie —

*Partie 4: Espace chromatique $L^*a^*b^*$ CIE 1976*



Reference number
ISO/CIE 11664-4:2019(E)

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Contents

	Page
Foreword	iv
Introduction	v
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
4 Symbols and abbreviated terms	2
5 Calculation method	2
5.1 Basic coordinates	2
5.2 Correlates of lightness, chroma and hue	4
5.3 Colour differences	4
Annex A (informative) Reverse transformation	7
Bibliography	8

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by the International Commission on Illumination (CIE) in cooperation with Technical Committee ISO/TC 274, *Light and lighting*.

This first edition of ISO/CIE 11664-4 cancels and replaces ISO 11664-4:2008 | CIE 11664-4:2007, of which it constitutes a minor revision. The document has been editorially revised as per current ISO rules and the references have been updated.

A list of all parts in the ISO 11664 and ISO/CIE 11664 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The three-dimensional colour space produced by plotting CIE tristimulus values (X , Y , Z) in rectangular coordinates is not visually uniform, nor is the (x, y, Y) space nor the two-dimensional CIE (x, y) chromaticity diagram. Equal distances in these spaces do not represent equally perceptible differences between colour stimuli. For this reason, in 1976, the CIE introduced and recommended two new spaces (known as CIELAB and CIELUV) whose coordinates are nonlinear functions of X , Y and Z . The recommendation was put forward in an attempt to unify the then very diverse practice in uniform colour spaces and associated colour-difference formulae^{[1][2]}. Both these more-nearly uniform colour spaces have become well accepted and widely used. Numerical values representing approximately the magnitude of colour differences can be described by simple Euclidean distances in the spaces or by more sophisticated formulae that improve the correlation with the perceived size of differences.

The purpose of this document is to define procedures for calculating the coordinates of the CIE 1976 $L^*a^*b^*$ (CIELAB) colour space and the Euclidean colour difference values based on these coordinates. This document does not cover more sophisticated colour-difference formulae based on CIELAB, such as the CMC formula^[3], the CIE94 formula^[4], the DIN99 formula^[5], and the CIEDE2000 formula^{[6][7]}, nor does it cover the alternative uniform colour space, CIELUV^[8].

Colorimetry —

Part 4: CIE 1976 L*a*b* colour space

1 Scope

This document specifies a method of calculating the coordinates of the CIE 1976 L*a*b* colour space, including correlates of lightness, chroma and hue. It includes two methods for calculating Euclidean distances in this space to represent the perceived magnitude of colour differences.

This document is applicable to tristimulus values calculated using colour-matching functions of the CIE 1931 standard colorimetric system or the CIE 1964 standard colorimetric system. This document can be used for the specification of colour stimuli perceived as belonging to a reflecting or transmitting object, where a three-dimensional space more uniform than tristimulus space is required.

This document does not apply to colour stimuli perceived as belonging to an area that appears to be emitting light as a primary light source, or that appears to be specularly reflecting such light.

This document is applicable to self-luminous displays, such as cathode ray tubes, if they are being used to simulate reflecting or transmitting objects and if the stimuli are appropriately normalized.

Calculating the reverse transformation is shown in [Annex A](#).

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

CIE S 017, *ILV: International Lighting Vocabulary*

ISO/CIE 11664-1, *Colorimetry Part 1 — CIE Standard Colorimetric Observers*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in CIE S 017 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

4 Symbols and abbreviated terms

X, Y, Z	tristimulus values of test stimulus calculated using the colour-matching functions of the CIE 1931 standard colorimetric system (also known as the CIE 2° standard colorimetric system)
X_n, Y_n, Z_n	tristimulus values of a specific white colour stimulus calculated using the colour-matching functions of the CIE 1931 standard colorimetric system
L^*	CIE 1976 lightness (CIELAB lightness)
a^*, b^*	CIELAB a^*, b^* coordinates
C_{ab}^*	CIE 1976 a,b chroma (CIELAB chroma)
h_{ab}	CIE 1976 a,b hue angle (CIELAB hue angle)
ΔL^*	CIELAB lightness difference
$\Delta a^*, \Delta b^*$	CIELAB a^*, b^* difference
ΔC_{ab}^*	CIELAB chroma difference
Δh_{ab}	CIELAB hue angle difference
ΔH_{ab}^*	CIELAB hue difference
ΔE_{ab}^*	CIELAB colour difference

If the character “Δ” is not available, it may be replaced by the character “D”.

The phrase “CIE 1976 $L^*a^*b^*$ ” and the term “CIELAB” may be used interchangeably.

Where tristimulus values are calculated using the colour-matching functions of the CIE 1964 standard colorimetric system (also known as the CIE 10° standard colorimetric system), a subscript 10 shall be added to all the above symbols.

5 Calculation method

5.1 Basic coordinates

The CIE 1976 $L^*a^*b^*$ colour space is a three-dimensional, approximately uniform colour space produced by plotting in rectangular coordinates, L^*, a^*, b^* , quantities defined by [Formulae \(1\) to \(3\)](#):

$$L^* = 116 f(Y/Y_n) - 16 \quad (1)$$

$$a^* = 500 [f(X/X_n) - f(Y/Y_n)] \quad (2)$$

$$b^* = 200 [f(Y/Y_n) - f(Z/Z_n)] \quad (3)$$

where

$$f(X/X_n) = (X/X_n)^{1/3} \quad \text{if } (X/X_n) > (6/29)^3 \quad (4)$$

$$f(X/X_n) = (841/108)(X/X_n) + 4/29 \quad \text{if } (X/X_n) \leq (6/29)^3 \quad (5)$$

and

$$f(Y/Y_n) = (Y/Y_n)^{1/3} \quad \text{if } (Y/Y_n) > (6/29)^3 \quad (6)$$

$$f(Y/Y_n) = (841/108)(Y/Y_n) + 4/29 \quad \text{if } (Y/Y_n) \leq (6/29)^3 \quad (7)$$

and

$$f(Z/Z_n) = (Z/Z_n)^{1/3} \quad \text{if } (Z/Z_n) > (6/29)^3 \quad (8)$$

$$f(Z/Z_n) = (841/108)(Z/Z_n) + 4/29 \quad \text{if } (Z/Z_n) \leq (6/29)^3 \quad (9)$$

where

X, Y, Z are the tristimulus values of the test colour stimulus based on the CIE 1931 standard colorimetric system defined in ISO/CIE 11664-1;

X_n, Y_n, Z_n are the corresponding tristimulus values of a specified white stimulus.

In cases of simulated reflecting or transmitting objects produced on a self-luminous display, all the tristimulus values shall be first normalized by the same factor so that Y would be equal to 100 for an object with 100 % reflectance or transmittance.

If the angle subtended at the eye by the test stimulus is between about 1° and 4° , the tristimulus values X, Y, Z calculated using the colour-matching functions of the CIE 1931 standard colorimetric system should be used. If this angular subtense is greater than 4° , the tristimulus values X_{10}, Y_{10}, Z_{10} calculated using the colour-matching functions of the CIE 1964 standard colorimetric system should be used. The same colour-matching functions and the same specified white stimulus shall be used for all stimuli to be compared with each other.

When tristimulus values based on the CIE 1964 standard colorimetric system defined in ISO/CIE 11664-1 are used, a subscript 10 shall be added to all the symbols in [Formulae \(1\) to \(9\)](#).

If the tristimulus values X, Y, Z are obtained by spectrophotometry, the tristimulus values X_n, Y_n, Z_n of the specified white stimulus shall be calculated using the same method as used for the test stimulus (same colour-matching functions, same range and interval of wavelength, and same bandwidth). If the tristimulus values X, Y, Z are obtained by direct measurement using a tristimulus colorimeter, X_n, Y_n, Z_n shall be measured using the same tristimulus colorimeter and a white reflectance standard calibrated relative to a perfect reflecting diffuser.

NOTE 1 For real object colours, the specified white stimulus normally chosen for X_n, Y_n, Z_n is light reflected from a perfect reflecting diffuser illuminated by the same light source as the test object. In this case, X_n, Y_n, Z_n are the tristimulus values of the light source normalized by a common factor so that Y_n is equal to 100. For simulated object colours, the specified white stimulus normally chosen is one that has the appearance of a perfect reflecting diffuser, again normalized by a common factor so that Y_n is equal to 100.

NOTE 2 Examples of values of X_n, Y_n and Z_n for specific illuminants and specific calculation methods have been published^[2].

NOTE 3 [Formulae \(5\), \(7\) and \(9\)](#) are based on a suggestion by Pauli^[2].

NOTE 4 A value of 7,787 is approximately equal to the term (841/108) in [Formulae \(5\), \(7\) and \(9\)](#). The approximate value can be used in practice.

NOTE 5 A value of 0,008 856 is approximately equal to the term (6/29)³ in [Formulae \(4\), \(5\), \(6\), \(7\), \(8\) and \(9\)](#). The approximate value can be used in practice.

NOTE 6 The fractions 6/29 and 4/29 in [Formulae \(4\) to \(9\)](#) are exactly equal to the fractions 24/116 and 16/116 appearing in CIE 15^[2].

NOTE 7 The term (841/108) in [Formulae \(5\), \(7\) and \(9\)](#) is derived from and exactly equal to (1/3) × (29/6)².

NOTE 8 [Formula \(1\)](#) reduces to $L^* \approx 903,3(Y/Y_n)$ when $Y/Y_n \leq (6/29)^3$.

When CIELAB values are reported, they should be accompanied by all relevant information relating to the measurement conditions and the procedures used to calculate the input tristimulus values.

5.2 Correlates of lightness, chroma and hue

Approximate correlates of the perceived attributes lightness, chroma and hue shall be calculated according to [Formulae \(10\) and \(11\)](#):

CIE 1976 lightness (CIELAB lightness): L^* as defined in [5.1](#).

$$\text{CIE 1976 a,b chroma (CIELAB chroma): } C_{ab}^* = \left[(a^*)^2 + (b^*)^2 \right]^{1/2} \quad (10)$$

$$\text{CIE 1976 a,b hue angle (CIELAB hue angle): } h_{ab} = \arctan(b^*/a^*) \quad (11)$$

CIELAB hue angle, h_{ab} , shall lie between 0° and 90° if a^* and b^* are both positive, between 90° and 180° if b^* is positive and a^* is negative, between 180° and 270° if b^* and a^* are both negative, and between 270° and 360° if b^* is negative and a^* is positive.

NOTE When one of the linear [Formulae \(5\), \(7\) or \(9\)](#) is used for X/X_n , Y/Y_n or Z/Z_n , anomalous values of h_{ab} could be obtained^[10]. Anomalous values are unlikely to occur for reflecting object colours but could occur for transparent object colours of low luminance factor lying close to the spectrum locus or purple line.

5.3 Colour differences

Euclidean distances in CIELAB colour space can be used to represent approximately the perceived magnitude of colour differences between object colour stimuli of approximately the same size, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field with the chromaticity of CIE standard illuminant D65 defined in ISO 11664-2 | CIE S 014-2. The values given by this document may not correlate well with perceived colour differences in other viewing conditions.

Differences between two stimuli denoted by subscripts 0 (usually the reference) and 1 (usually the test) shall be calculated according to [Formulae \(12\) to \(17\)](#):

$$\Delta L^* = L_1^* - L_0^* \quad (12)$$

$$\Delta a^* = a_1^* - a_0^* \quad (13)$$

$$\Delta b^* = b_1^* - b_0^* \quad (14)$$

$$\Delta C_{ab}^* = C_{ab,1}^* - C_{ab,0}^* \quad (15)$$

$$\Delta h_{ab} = h_{ab,1} - h_{ab,0} \quad (16)$$

$$\Delta H_{ab}^* = 2 \left(C_{ab,1}^* \cdot C_{ab,0}^* \right)^{1/2} \sin(\Delta h_{ab} / 2) \quad (17)$$

For small colour differences away from the achromatic axis $\Delta C_{ab}^* = 0$, [Formula \(17\)](#) reduces to [Formula \(18\)](#):

$$\Delta H_{ab}^* \approx \left(C_{ab,1}^* \cdot C_{ab,0}^* \right)^{1/2} \Delta h_{ab} \quad (18)$$

where the value of Δh_{ab} is in radians.

If the line joining the two colours crosses the positive a^* axis, [Formula \(16\)](#) will give a value outside the range $\pm 180^\circ$. In this case, the value of Δh_{ab} shall be corrected by adding or subtracting 360° to bring it within this range.

NOTE 1 The quantity ΔH_{ab}^* is introduced to provide congruence with the perceptual understanding that a colour difference can be divided into a vector sum of a lightness difference, a chroma difference and a hue difference.

NOTE 2 The division of CIELAB colour differences into hue and chroma differences is progressively less useful as the absolute value of Δh_{ab} approaches 180° .

NOTE 3 In information technology and other fields, the subscripts r (for reference) and t (for test) are sometimes used instead of 0 and 1, respectively. Similarly, in industrial evaluations of small colour differences, s (for standard) and b (for batch) are sometimes used. In other applications, std (for standard) and spl (for sample) are sometimes used.

The CIE 1976 a, b colour difference, ΔE_{ab}^* , between two colour stimuli is calculated as the Euclidean distance between the points representing them in the space according to [Formulae \(19\)](#) or [\(20\)](#):

$$\Delta E_{ab}^* = \left[(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2 \right]^{1/2} \quad (19)$$

or

$$\Delta E_{ab}^* = \left[(\Delta L^*)^2 + (\Delta C_{ab}^*)^2 + (\Delta H_{ab}^*)^2 \right]^{1/2} \quad (20)$$

These two definitions of ΔE_{ab}^* are equivalent.

Other ways of calculating ΔH_{ab}^* are according to [Formula \(21\)](#):

$$\Delta H_{ab}^* = \left[(\Delta E_{ab}^*)^2 - (\Delta L^*)^2 - (\Delta C_{ab}^*)^2 \right]^{1/2} \quad (21)$$

where

ΔE_{ab}^* is calculated from [Formula \(19\)](#);

ΔH_{ab}^* has the same sign as Δh_{ab} .

$$\Delta H_{ab}^* = k \left[2 \left(C_{ab,1}^* \cdot C_{ab,0}^* - a_1^* \cdot a_0^* - b_1^* \cdot b_0^* \right) \right]^{1/2} \quad (22)$$

where $k = -1$ if $a_1^* \cdot b_0^* \geq a_0^* \cdot b_1^*$, otherwise $k = 1$;

and

$$\Delta H_{ab}^* = \left(a_0^* \cdot b_1^* - a_1^* \cdot b_0^* \right) / \left[0,5 \left(C_{ab,1}^* \cdot C_{ab,0}^* + a_1^* \cdot a_0^* + b_1^* \cdot b_0^* \right) \right]^{1/2} \quad (23)$$

NOTE 4 [Formula \(23\)](#) cannot be used when either of the compared chromas is zero and is imprecise when either chroma is close to zero.

NOTE 5 More details on the various methods of calculating ΔH_{ab}^* are given by Sève^[11] for [Formula \(17\)](#), by Stokes and Brill^[12] for [Formula \(22\)](#), and by Sève^[13] for [Formula \(23\)](#).

In different practical applications, it may be necessary to modify [Formula \(20\)](#) using different weightings for ΔL^* , ΔC_{ab}^* and ΔH_{ab}^* to obtain better correlation with colour-difference perception. In 2001, the CIE recommended such weightings in a new formula for the industrial evaluation of small colour differences^{[6][7]}, but this is outside the scope of this document. A colour difference obtained in this way is not a CIELAB colour difference and the symbol ΔE_{ab}^* should not be used.

Annex A (informative)

Reverse transformation

Formulae (A.1) to (A.9) represent the reverse transformation, i.e. the calculation of X , Y , Z when L^* , a^* , b^* are given:

$$f(Y/Y_n) = (L^* + 16) / 116 \quad (\text{A.1})$$

$$f(X/X_n) = a^* / 500 + f(Y/Y_n) \quad (\text{A.2})$$

$$f(Z/Z_n) = f(Y/Y_n) - b^* / 200 \quad (\text{A.3})$$

$$X = X_n [f(X/X_n)]^3 \quad \text{if } f(X/X_n) > 6/29 \quad (\text{A.4})$$

$$X = (108/841) X_n [f(X/X_n) - 4/29] \quad \text{if } f(X/X_n) \leq 6/29 \quad (\text{A.5})$$

$$Y = Y_n [f(Y/Y_n)]^3 \quad \text{if } f(Y/Y_n) > 6/29 \quad (\text{A.6})$$

$$Y = (108/841) Y_n [f(Y/Y_n) - 4/29] \quad \text{if } f(Y/Y_n) \leq 6/29 \quad (\text{A.7})$$

$$Z = Z_n [f(Z/Z_n)]^3 \quad \text{if } f(Z/Z_n) > 6/29 \quad (\text{A.8})$$

$$Z = (108/841) Z_n [f(Z/Z_n) - 4/29] \quad \text{if } f(Z/Z_n) \leq 6/29 \quad (\text{A.9})$$

NOTE 1 The condition in Formula (A.6) is equivalent to $L^* > 8$.

NOTE 2 The condition in Formula (A.7) is equivalent to $L^* \leq 8$.

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